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A NOVEL SHEAR DEFORMATION THEORY FOR MAGNETO-ELECTRO-ELASTIC NANOPLATES

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Abstract: This study proposes an innovative and efficient theoretical model for analyzing magneto-electro-elastic (MEE) nanoplates. A novel approach, formulated using third- and fifth-order Chebyshev polynomials based on Eringen's theory and the shear deformation theory, is developed. This formulation automatically enforces the zero-shear-stress condition at the plate's top and bottom surfaces, eliminating the need for any supplementary constraints. Applying the principle of extended virtual displacement, the formulation is developed in weak form. By integrating a nonlocal isogeometric analysis, the proposed model effectively captures small-scale effects in MEE nanoplate structures. The natural frequencies of the MEE nanoplate are investigated with respect to geometric and a nonlocal parameter. Comparative numerical results verify the reliability and superior performance of the proposed theory over existing higher-order shear deformation models.

Keywords: Chebyshev polynomials; magneto-electro-elastic (MEE) materials; nonlocal effects; nanostructures; isogeometric analysis (IGA).

1. INTRODUCTION

Owing to their inherent coupled mechanical, electrical and magnetic behaviors, magneto-electro-elastic (MEE) materials have become prominent candidates for various advanced technological applications in sensor, advanced aerospace, robotic, wearable devices, etc., [1-3]. At the nanoscale, their mechanical responses become size-dependent, which classical elasticity theory cannot describe due to the absence of a characteristic material lengh scale. To overcome this limitation, several generalized continuum theories: couple stress, nonlocal, strain gradient and surface elasticity theories were proposed. Among these, Eringen's theory is widely used, as it accounts for scale effects by relating stress at a point to strains across the entire domain. This theory has proved effective in

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modeling buckling, vibration and static of nanostructures.

Within this framework, numerous studies have addressed mechanical behaviors of MEE nanostructures. Using analytical solutions, Ke et al. [4] studied free vibration of MEE rectangular nanoplates, while Jafarsadeghi et al. [5] investigated circular nanoplates. Size-dependent analysis of MEE nanoplates resting on a foundation was examined by Jamalpoor et al. [6]. Liu et al. [7] examined vibration of piezoelectric nanoplates using Kirchhoff plate theories. Employing the classical plate theory and the nonlocal strain gradient framework [8] addressed vibration and buckling of porous MEE nanoplates. Mohammadimehr and Rostami [9] utilized the analytical solution to perform structural responses of the MEE nanoplates. Zur et al. [10] adopted the sinusoidal shear deformation theory to evaluate the critical load and natural frequency of MEE nanoplates. Gholami and Ansari [11] studied postbuckling behavior of MEE nanoplates based on analytical methods. Moreover, Malikan and Nguyen [12] examined hygrothermal buckling analysis of MEE nanoplates through a one-variable plate theory. Subsequent advancements of the nonlocal strain gradient framework for MEE nanoplates were presented in [13-15], whereas small-scale effects on MEE microplates were also discussed in [16, 17].

Despite these significant contributions, the coupling of nonlocal elasticity theory with isogeometric analysis for MEE nanostructures has yet to be addressed in the open literature. The present study aims to fill this gap by proposing a novel computational approach that integrates third- and fifth-order Chebyshev polynomials-based shear deformation theories within Eringen's nonlocal elasticity framework using isogeometric analysis scheme for the analyis of MEE nanoplates. It is also noteworthy that the Chebyshev shear deformation theory has been previous applied to plate structures [18-20], though without accounting for size-dependent effects. Compared with available formulations, a notable advantage of the proposed approach is that the zero-shear-stress condition on the plate's top and bottom faces is satisfied inherently, with no supplementary constraints required. Therefore, the proposed formulation not only extends the applicability of nonlocal elasticity within the IGA but also introduces an innovative perspective for accurate modelling of MEE nanostructures at small scales.

2. NONLOCAL ANALYSIS OF MEE NANOPLATES

According to the special Helmholtz averaging kernel, $L=\left(1-\xi^2\nabla^2\right)$, and Eringen's elasticity theory [21], basic equations for MEE structures are expressed

$$
\begin{aligned}
&\left(1-\xi^2\nabla^2\right)t_{ij}=c_{ijkl}\varepsilon_{kl}-e_{ijk}E_k-q_{kij}H_k\\
&\left(1-\xi^2\nabla^2\right)\bar{D}_i=e_{ikl}\varepsilon_{kl}-k_{ik}E_k-d_{ik}H_k\\
&\left(1-\xi^2\nabla^2\right)\bar{B}_i=q_{ikl}\varepsilon_{kl}-d_{ik}E_k-\mu_{ik}H_k\\
&\left(1-\xi^2\nabla^2\right)t_{ij}=\sigma_{ij};\ \left(1-\xi^2\nabla^2\right)\bar{D}_i=D_i;\ \left(1-\xi^2\nabla^2\right)\bar{B}_i=B_i\\
&\sigma_{ij,j}=\left(1-\xi^2\nabla^2\right)\rho\ddot{u}_i;\ B_{i,i}=0;\ D_{i,i}=0
\end{aligned}
\tag{1}
$$

where ξ is scale parameter for size effect; σ_{ij}, D_i and B_i represent local stress, electric displacement and magnetic induction components, respectively; t_{ij}, $\bar{D}_i$ and $\bar{B}_i$ are the nonlocal stress, electric displacement and magnetic induction components, respectively; ρ and u_i are mass density and displacement components, respectively; $c_{ij}, e_{ij}, q_{ij}, k_{ij}, d_{ij}, \mu_{ij}$ are piezoelectric, piezo-magnetic, dielectric permittivity, electromagnetic, magnetic permittivity coefficients, respectively; ε_{ij}, E_i and H_i represent strain, electric potential and magnetic potential components, respectively.

Based on the principle of extended virtual displacements, the equations of motion for the MEE structures are obtained

$$
\int_V \sigma_{ij,j}\delta u_i \mathrm{d}V+\int_V D_{i,i}\delta\Phi_i \mathrm{d}V+\int_V B_{i,i}\delta\Psi_i \mathrm{d}V+\int_V\left(1-\xi^2\nabla^2\right)\rho\ddot{u}_i\delta u_i \mathrm{d}V=0 \tag{2}
$$

where $\delta\Psi_i$, $\delta\Phi_i$ and δu_i are the magnetic potentials, electric and virtual displacement, respectively.

Applying the divergence theorem while omitting the Neumann boundary traction yields

$$
\int_V \delta\left(\varepsilon_{ij}\right)^T\sigma_{ij}\mathrm{d}V-\int_V \delta\left(E_i\right)^T D_i \mathrm{d}V-\int_V \delta\left(H_i\right)^T B_i \mathrm{d}V+\int_V\left(1-\xi^2\nabla^2\right)\delta\left(u_i\right)^T\rho\ddot{u}_i \mathrm{d}V=0 \tag{3}
$$

where $\delta u_{i,j}=\delta\varepsilon_{ij}$; $\delta\Phi_{i,i}=-\delta E_i$; $\delta\Psi_{i,i}=-\delta H_i$.

Eq. (3) can be rewritten in compact form as

$$
\delta U+\delta K=0 \tag{4}
$$

where

$$
\begin{aligned}
\delta U&=\int_V \delta\left(\varepsilon_{ij}\right)^T\sigma_{ij}\mathrm{d}V-\int_V \delta\left(E_i\right)^T D_i \mathrm{d}V-\int_V \delta\left(H_i\right)^T B_i \mathrm{d}V\\
\delta K&=\int_V\left(1-\xi^2\nabla^2\right)\delta\left(u_i\right)^T\rho\ddot{u}_i \mathrm{d}V
\end{aligned}
\tag{5}
$$

in which δU and δK are the virtual strain energy and virtual kinetic energy, respectively.

3. DISPLACEMENT FIELDS

In according with [22], displacement fields are given as

$$
\begin{Bmatrix}\bar{u}\\ \bar{v}\\ \bar{w}\end{Bmatrix}=\begin{Bmatrix}u_0\\ v_0\\ w\end{Bmatrix}-z\begin{Bmatrix}\beta_x\\ \beta_y\\ 0\end{Bmatrix}+f(z)\begin{Bmatrix}\theta_x\\ \theta_y\\ 0\end{Bmatrix} \tag{6}
$$

where θ_x and θ_y represent rotations; u, v and w are the displacement components; $\beta_x = w_{,x}$ and $\beta_y = w_{,y}$; $f(z)$ is known as a transverse shear function, which defines the distribution shear stresses across the thickness, assuming that the top and bottom surfaces are free of shear stress. In the present work, a unified formula [18-20] is employed, in which Chebyshev polynomials of various orders are utilized, as detailed in Table 1, where the parameter p rdenotes the polynomial order. Figure 1 shows the Chebyshev polynomial and its derivation. It can be observed that the zero-shear-stress requirement on the plate's top and bottom faces is inherently fulfilled within the proposed formulation without the introduction of extra conditions.

Table 1. The Chebyshev function.

p=3 (3rd)	$f(z) = \cos(3\times\cos^{-1}(z/h))$
p=5 (5th)	$f(z) = \cos(5\times\cos^{-1}(z/h)) + \frac{5}{h}z$

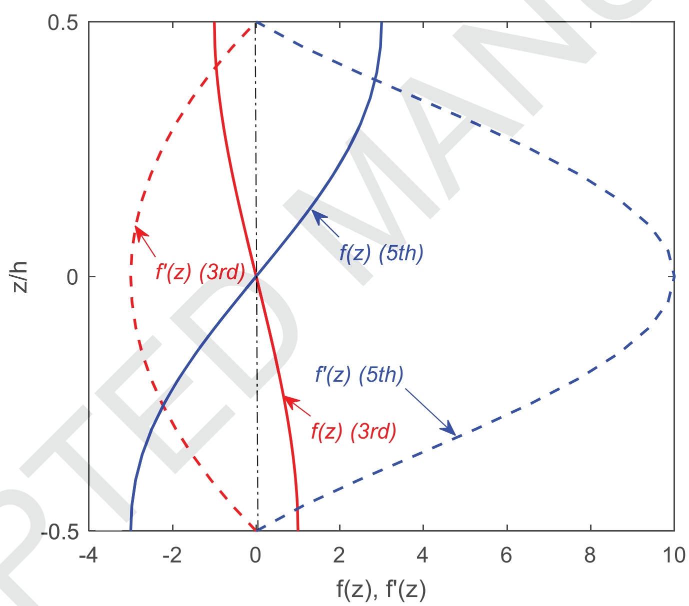


Figure 1. Chebyshev polynomial and its derivation.

Based on Eq. (6), the strains can be written as

$$\boldsymbol{\gamma} = \{\gamma_{xz} \quad \gamma_{yz}\}^{\mathrm{T}} = \boldsymbol{\varepsilon}_{s0} + f'(z)\boldsymbol{\varepsilon}_{s1}; \quad \boldsymbol{\varepsilon} = \{\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy}\}^{\mathrm{T}} = \boldsymbol{\varepsilon}_{b0} + z\boldsymbol{\varepsilon}_{b1} + f(z)\boldsymbol{\varepsilon}_{b2} \tag{7}$$

where $f'(z)$ is derivative of $f(z)$ and

$$\boldsymbol{\varepsilon}_{b0} = \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{Bmatrix}; \ \boldsymbol{\varepsilon}_{b1} = -\begin{Bmatrix} \beta_{x,x} \\ \beta_{y,y} \\ \beta_{x,y} + \beta_{y,x} \end{Bmatrix}; \ \boldsymbol{\varepsilon}_{b2} = \begin{Bmatrix} \theta_{x,x} \\ \theta_{y,y} \\ \theta_{x,y} + \theta_{y,x} \end{Bmatrix}; \ \boldsymbol{\varepsilon}_{s0} = \begin{Bmatrix} w_{,x} - \beta_x \\ w_{,y} - \beta_y \end{Bmatrix}; \ \boldsymbol{\varepsilon}_{s1} = \begin{Bmatrix} \theta_x \\ \theta_y \end{Bmatrix} \tag{8}$$

The present formulation employs electric and magnetic potentials consistent with those in [4]

$$
\begin{aligned}
\Phi(x,y,z,t) &= -\cos\left(\frac{\pi z}{h}\right)\varphi(x,y,t) + \frac{2z}{h}V_0 \\
\Psi(x,y,z,t) &= -\cos\left(\frac{\pi z}{h}\right)\psi(x,y,t) + \frac{2z}{h}\Omega_0
\end{aligned} \tag{9}
$$

where V_0 is the electric voltage and Ω_0 is the magnetic potential.

Magnetic and electric potential components can be expressed

$$
\begin{aligned}
\mathbf{E} &= \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} = -\begin{Bmatrix} \Phi_{,x} \\ \Phi_{,y} \\ \Phi_{,z} \end{Bmatrix} = -\begin{Bmatrix} -\cos\left(\frac{\pi z}{h}\right)\varphi_{,x} \\ -\cos\left(\frac{\pi z}{h}\right)\varphi_{,y} \\ \frac{\pi}{h}\sin\left(\frac{\pi z}{h}\right)\varphi + \frac{2V_0}{h} \end{Bmatrix} \\
\mathbf{H} &= \begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix} = -\begin{Bmatrix} \Psi_{,x} \\ \Psi_{,y} \\ \Psi_{,z} \end{Bmatrix} = -\begin{Bmatrix} -\cos\left(\frac{\pi z}{h}\right)\psi_{,x} \\ -\cos\left(\frac{\pi z}{h}\right)\psi_{,y} \\ \frac{\pi}{h}\sin\left(\frac{\pi z}{h}\right)\psi + \frac{2\Omega_0}{h} \end{Bmatrix}
\end{aligned} \tag{10}
$$

δU in Eq. (4) can be reformulated as follows

$$
\begin{aligned}
\delta U = &\int_\Omega \delta(\bar{\boldsymbol{\varepsilon}}_b)^T \left(\bar{\mathbf{D}}^b_{uu}\bar{\boldsymbol{\varepsilon}}_b - \bar{\mathbf{D}}^b_{ue}\bar{\mathbf{E}}_b - \bar{\mathbf{D}}^b_{um}\bar{\mathbf{H}}_b\right)\mathrm{d}\Omega + \int_\Omega \delta(\bar{\boldsymbol{\varepsilon}}_s)^T \left(\mathbf{D}^s_{uu}\bar{\boldsymbol{\varepsilon}}_s - \bar{\mathbf{D}}^s_{ue}\mathbf{E}_s - \bar{\mathbf{D}}^s_{um}\bar{\mathbf{H}}_s\right)\mathrm{d}\Omega \\
&-\int_\Omega \delta(\bar{\mathbf{E}}_b)^T \left(\bar{\mathbf{D}}^b_{eu}\bar{\boldsymbol{\varepsilon}}_b + \bar{\mathbf{D}}^b_{ee}\bar{\mathbf{E}}_b + \bar{\mathbf{D}}^b_{em}\bar{\mathbf{H}}_b\right)\mathrm{d}\Omega - \int_\Omega \delta(\bar{\mathbf{E}}_s)^T \left(\bar{\mathbf{D}}^s_{eu}\bar{\boldsymbol{\varepsilon}}_s + \bar{\mathbf{D}}^s_{ee}\bar{\mathbf{E}}_s + \bar{\mathbf{D}}^s_{em}\bar{\mathbf{H}}_s\right)\mathrm{d}\Omega \\
&-\int_\Omega \delta(\bar{\mathbf{H}}_b)^T \left(\bar{\mathbf{D}}^b_{mu}\bar{\boldsymbol{\varepsilon}}_b + \bar{\mathbf{D}}^b_{me}\bar{\mathbf{E}}_b + \bar{\mathbf{D}}^b_{mm}\bar{\mathbf{H}}_b\right)\mathrm{d}\Omega - \int_\Omega \delta(\bar{\mathbf{H}}_s)^T \left(\bar{\mathbf{D}}^s_{mu}\bar{\boldsymbol{\varepsilon}}_s + \bar{\mathbf{D}}^s_{me}\bar{\mathbf{E}}_s + \bar{\mathbf{D}}^s_{mm}\bar{\mathbf{H}}_s\right)\mathrm{d}\Omega
\end{aligned} \tag{11}
$$

where

$$
\begin{aligned}
&\bar{\boldsymbol{\varepsilon}}_b = \begin{Bmatrix} \boldsymbol{\varepsilon}_{b0} \\ \boldsymbol{\varepsilon}_{b1} \\ \boldsymbol{\varepsilon}_{b2} \end{Bmatrix};\ \bar{\boldsymbol{\varepsilon}}_s = \begin{Bmatrix} \boldsymbol{\varepsilon}_{s0} \\ \boldsymbol{\varepsilon}_{s1} \end{Bmatrix};\ \bar{\mathbf{E}}_b = -\begin{Bmatrix} 0 \\ 0 \\ \varphi \end{Bmatrix};\ \bar{\mathbf{E}}_s = -\begin{Bmatrix} \varphi_{,x} \\ \varphi_{,y} \end{Bmatrix};\ \bar{\mathbf{H}}_b = -\begin{Bmatrix} 0 \\ 0 \\ \psi \end{Bmatrix};\ \bar{\mathbf{H}}_s = -\begin{Bmatrix} \psi_{,x} \\ \psi_{,y} \end{Bmatrix} \\
&\bar{\mathbf{D}}^b_{uu} = \begin{bmatrix} \mathbf{A}^b & \mathbf{B}^b & \mathbf{E}^b \\ \mathbf{B}^b & \mathbf{D}^b & \mathbf{F}^b \\ \mathbf{E}^b & \mathbf{F}^b & \mathbf{H}^b \end{bmatrix};\ \mathbf{D}^s_{uu} = \begin{bmatrix} \mathbf{A}^s & \mathbf{B}^s \\ \mathbf{B}^s & \mathbf{D}^s \end{bmatrix} \\
&\left(\mathbf{A}^b, \mathbf{B}^b, \mathbf{D}^b, \mathbf{E}^b, \mathbf{F}^b, \mathbf{H}^b\right) = \int_{-h/2}^{h/2} \left(1, z, z^2, f(z), z f(z), f^2(z)\right)\mathbf{C}_{uub}\mathrm{d}z \\
&\left(\mathbf{A}^s, \mathbf{B}^s, \mathbf{D}^s\right) = \int_{-h/2}^{h/2} \left(1, f'(z), f'^2(z)\right)\mathbf{C}_{uus}\mathrm{d}z
\end{aligned} \tag{12}
$$

$$\bar{\mathbf{D}}_{ue}^{b}=\left\{\hat{\mathbf{C}}_{ueb}^{0}\quad \hat{\mathbf{C}}_{ueb}^{1}\quad \hat{\mathbf{C}}_{ueb}^{2}\right\}^{T};\ \left(\hat{\mathbf{C}}_{ueb}^{0},\hat{\mathbf{C}}_{ueb}^{1},\hat{\mathbf{C}}_{ueb}^{2}\right)=-\int_{-h/2}^{h/2}\mathbf{C}_{ueb}\left(1,z,f\left(z\right)\right)\frac{\pi}{h}\sin\left(\frac{\pi z}{h}\right)\mathrm{d}z$$

$$\bar{\mathbf{D}}_{um}^{b}=\left\{\hat{\mathbf{C}}_{umb}^{0}\quad \hat{\mathbf{C}}_{umb}^{1}\quad \hat{\mathbf{C}}_{umb}^{2}\right\}^{T};\ \left(\hat{\mathbf{C}}_{umb}^{0},\hat{\mathbf{C}}_{umb}^{1},\hat{\mathbf{C}}_{umb}^{2}\right)=-\int_{-h/2}^{h/2}\mathbf{C}_{umb}\left(1,z,f\left(z\right)\right)\frac{\pi}{h}\sin\left(\frac{\pi z}{h}\right)\mathrm{d}z$$

$$\bar{\mathbf{D}}_{ue}^{s}=\left\{\hat{\mathbf{C}}_{ues}^{0}\quad \hat{\mathbf{C}}_{ues}^{1}\right\}^{T};\ \left(\hat{\mathbf{C}}_{ues}^{0},\hat{\mathbf{C}}_{ues}^{1}\right)=-\int_{-h/2}^{h/2}\mathbf{C}_{ues}\left(1,f'\left(z\right)\right)\left(-\cos\left(\frac{\pi z}{h}\right)\right)\mathrm{d}z$$

$$\bar{\mathbf{D}}_{um}^{s}=\left\{\hat{\mathbf{C}}_{ums}^{0}\quad \hat{\mathbf{C}}_{ums}^{1}\right\}^{T};\ \left(\hat{\mathbf{C}}_{ums}^{0},\hat{\mathbf{C}}_{ums}^{1}\right)=-\int_{-h/2}^{h/2}\mathbf{C}_{ums}\left(1,f'\left(z\right)\right)\left(-\cos\left(\frac{\pi z}{h}\right)\right)\mathrm{d}z$$

$$\bar{\mathbf{D}}_{ee}^{b}=\int_{-h/2}^{h/2}\mathbf{C}_{eeb}\left(\frac{\pi}{h}\sin\left(\frac{\pi z}{h}\right)\right)^{2}\mathrm{d}z;\ \bar{\mathbf{D}}_{ee}^{s}=\int_{-h/2}^{h/2}\mathbf{C}_{ees}\left(-\cos\left(\frac{\pi z}{h}\right)\right)^{2}\mathrm{d}z$$

$$\bar{\mathbf{D}}_{em}^{b}=\int_{-h/2}^{h/2}\mathbf{C}_{emb}\left(\frac{\pi}{h}\sin\left(\frac{\pi z}{h}\right)\right)^{2}\mathrm{d}z;\ \bar{\mathbf{D}}_{em}^{s}=\int_{-h/2}^{h/2}\mathbf{C}_{ems}\left(-\cos\left(\frac{\pi z}{h}\right)\right)^{2}\mathrm{d}z$$

$$\bar{\mathbf{D}}_{mm}^{b}=\int_{-h/2}^{h/2}\mathbf{C}_{mmb}\left(\frac{\pi}{h}\sin\left(\frac{\pi z}{h}\right)\right)^{2}\mathrm{d}z;\ \bar{\mathbf{D}}_{mm}^{s}=\int_{-h/2}^{h/2}\mathbf{C}_{mms}\left(-\cos\left(\frac{\pi z}{h}\right)\right)^{2}\mathrm{d}z$$

$$\bar{\mathbf{D}}_{eu}^{b}=\bar{\mathbf{D}}_{ue}^{bT};\bar{\mathbf{D}}_{eu}^{s}=\bar{\mathbf{D}}_{ue}^{sT};\bar{\mathbf{D}}_{mu}^{b}=\bar{\mathbf{D}}_{um}^{bT};\bar{\mathbf{D}}_{mu}^{s}=\bar{\mathbf{D}}_{um}^{sT};\bar{\mathbf{D}}_{me}^{b}=\bar{\mathbf{D}}_{em}^{bT};\bar{\mathbf{D}}_{me}^{s}=\bar{\mathbf{D}}_{em}^{sT}$$

in which

$$\mathbf{C}_{uub}=\begin{bmatrix}\bar{c}_{11}&\bar{c}_{12}&0\\\bar{c}_{21}&\bar{c}_{22}&0\\0&0&\bar{c}_{66}\end{bmatrix};\mathbf{C}_{ueb}=\begin{bmatrix}0&0&\bar{e}_{31}\\0&0&\bar{e}_{32}\\0&0&0\end{bmatrix};\mathbf{C}_{umb}=\begin{bmatrix}0&0&\bar{q}_{31}\\0&0&\bar{q}_{32}\\0&0&0\end{bmatrix};\mathbf{C}_{eeb}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&\bar{k}_{33}\end{bmatrix}$$
$$\mathbf{C}_{uus}=\begin{bmatrix}\bar{c}_{55}&0\\0&\bar{c}_{44}\end{bmatrix};\ \mathbf{C}_{ues}=\begin{bmatrix}\bar{e}_{15}&0\\0&\bar{e}_{24}\end{bmatrix};\ \mathbf{C}_{ums}=\begin{bmatrix}\bar{q}_{15}&0\\0&\bar{q}_{24}\end{bmatrix};\mathbf{C}_{ees}=\begin{bmatrix}\bar{k}_{11}&0\\0&\bar{k}_{22}\end{bmatrix}\tag{13}$$
$$\mathbf{C}_{emb}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&\bar{d}_{33}\end{bmatrix};\ \mathbf{C}_{mmb}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&\bar{\mu}_{33}\end{bmatrix};\ \mathbf{C}_{ems}=\begin{bmatrix}\bar{d}_{11}&0\\0&\bar{d}_{22}\end{bmatrix};\ \mathbf{C}_{mms}=\begin{bmatrix}\bar{\mu}_{11}&0\\0&\bar{\mu}_{22}\end{bmatrix}$$

and [4]

$$\bar{c}_{11}=c_{11}-\frac{c_{13}^{2}}{c_{33}};\bar{c}_{12}=c_{12}-\frac{c_{13}^{2}}{c_{33}};\bar{c}_{66}=c_{66};\bar{c}_{55}=c_{55};\bar{c}_{44}=c_{44};\ \bar{e}_{31}=e_{31}-\frac{e_{33}c_{13}}{c_{33}};\ \bar{e}_{15}=e_{15}$$
$$\bar{q}_{31}=q_{31}-\frac{q_{33}c_{13}}{c_{33}};\ \bar{q}_{15}=q_{15};\ \bar{k}_{33}=k_{33}+\frac{e_{33}^{2}}{c_{33}};\bar{k}_{11}=k_{11};\bar{d}_{33}=d_{33}+\frac{q_{33}e_{33}}{c_{33}};\bar{d}_{11}=d_{11}\tag{14}$$
$$\bar{\mu}_{33}=\mu_{33}+\frac{q_{33}^{2}}{c_{33}};\bar{\mu}_{11}=\mu_{11}$$

δK in Eq. (4) is similarly formulated as

$$\delta K=\int_{\Omega}\left(1-\mu\nabla^{2}\right)\delta\mathbf{u}^{T}\mathbf{I}_{m}\ddot{\mathbf{u}}\mathrm{d}\Omega=\mathbf{0}\tag{15}$$

where

$$\bar{\mathbf{u}} = \begin{Bmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \mathbf{u}_2 \end{Bmatrix}; \mathbf{u}_0 = \begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix}; \mathbf{u}_1 = -\begin{Bmatrix} \beta_x \\ \beta_y \\ 0 \end{Bmatrix}; \mathbf{u}_2 = \begin{Bmatrix} \theta_x \\ \theta_y \\ 0 \end{Bmatrix}; \ \mathbf{I}_m = \begin{bmatrix} \mathbf{I}_1 & \mathbf{I}_2 & \mathbf{I}_4 \\ \mathbf{I}_2 & \mathbf{I}_3 & \mathbf{I}_5 \\ \mathbf{I}_4 & \mathbf{I}_5 & \mathbf{I}_6 \end{bmatrix} \tag{16}$$

$$\left(\mathbf{I}_1, \mathbf{I}_2, \mathbf{I}_3, \mathbf{I}_4, \mathbf{I}_5, \mathbf{I}_6\right) = \int_{-h/2}^{h/2} \rho\left(1, z, z^2, f(z), zf(z), f^2(z)\right)\mathbf{I}_{3\times3}\mathrm{d}z$$

where ρ is mass density.

The expression for the change in potential energy resulting from in-plane loading, including pre-buckling mechanical forces, is

$$\delta V = -h\int_{\Omega}\left(1-\xi^2\nabla^2\right)\delta\begin{Bmatrix} w_{,x} \\ w_{,y} \end{Bmatrix}^{\mathrm{T}}\begin{bmatrix} N_x^{mech}+N_x^{elec}+N_x^{mag} & 0 \\ 0 & N_y^{mech}+N_y^{elec}+N_y^{mag} \end{bmatrix}\begin{Bmatrix} w_{,x} \\ w_{,y} \end{Bmatrix}\mathrm{d}\Omega \tag{17}$$

or

$$\delta V = -h\int_{\Omega}\left(1-\xi^2\nabla^2\right)\delta\left(\mathbf{B}_g\right)^{\mathrm{T}}\begin{bmatrix} N_x^{mech}+N_x^{elec}+N_x^{mag} & 0 \\ 0 & N_y^{mech}+N_y^{elec}+N_y^{mag} \end{bmatrix}\mathbf{B}_g\mathrm{d}\Omega \tag{18}$$

where N_x^{mech} and N_y^{mech} indicate in-plane mechanical loads, and $N_x^{elec} = N_y^{elec} = 2\bar{e}_{31}V_0$; $N_x^{mag} = N_y^{mag} = 2\bar{q}_{31}\Omega_0$ [6, 12].

Consequently, equations of motion in Eq. (4) are written

$$\delta U + \delta K - \delta V = 0 \tag{19}$$

4. NURBS BASIS FUNCTIONS

Using NURBS basis functions (N_I), the displacement field in Eq. (6) is represented [23]

$$\mathbf{u}^h\left(x, y\right) = \sum_{I=1}^{m\times n} N_I\left(x, y\right)\mathbf{I}_{9\times9}\mathbf{q}_I \tag{20}$$

where $\mathbf{q}_I = \left\{u_I \quad v_I \quad w_I \quad \theta_{xI} \quad \theta_{yI} \quad \beta_{xI} \quad \beta_{yI} \quad \varphi_I \quad \psi_I\right\}^T$.

Substituting Eq. (20) into Eq. (8), we have

$$\begin{aligned} \bar{\boldsymbol{\varepsilon}}_b &= \left\{\boldsymbol{\varepsilon}_{b0} \quad \boldsymbol{\varepsilon}_{b1} \quad \boldsymbol{\varepsilon}_{b2}\right\}^T = \sum_{I=1}^{m\times n}\left\{\mathbf{B}_{b0I} \quad \mathbf{B}_{b1I} \quad \mathbf{B}_{b2I}\right\}^T\mathbf{q}_I = \sum_{I=1}^{m\times n}\bar{\mathbf{B}}_{bI}\mathbf{q}_I \\ \bar{\boldsymbol{\varepsilon}}_s &= \left\{\boldsymbol{\varepsilon}_{s0} \quad \boldsymbol{\varepsilon}_{s1}\right\}^T = \sum_{I=1}^{m\times n}\left\{\mathbf{B}_{s0I} \quad \mathbf{B}_{s1I}\right\}^T\mathbf{q}_I = \sum_{I=1}^{m\times n}\bar{\mathbf{B}}_{sI}\mathbf{q}_I \end{aligned} \tag{21}$$

where

$$\mathbf{B}_{b0I}=\begin{bmatrix} N_{I,x} & 0 & 0\,0\,0\,0\,0\,0\,0 \\ 0 & N_{I,y} & 0\,0\,0\,0\,0\,0\,0 \\ N_{I,y} & N_{I,x} & 0\,0\,0\,0\,0\,0\,0 \end{bmatrix};\ \mathbf{B}_{b1I}=-\begin{bmatrix} 0\,0\,0\,0\,0\,0\,0 & N_{I,x} & 0 & 0\,0 \\ 0\,0\,0\,0\,0\,0\,0 & 0 & N_{I,y} & 0\,0 \\ 0\,0\,0\,0\,0\,0\,0 & N_{I,y} & N_{I,x} & 0\,0 \end{bmatrix} \tag{22}$$

$$\mathbf{B}_{b2I}=\begin{bmatrix} 0\,0\,0 & N_{I,x} & 0 & 0\,0\,0\,0 \\ 0\,0\,0 & 0 & N_{I,y} & 0\,0\,0\,0 \\ 0\,0\,0 & N_{I,y} & N_{I,x} & 0\,0\,0\,0 \end{bmatrix}$$

$$\mathbf{B}_{s0I}=\begin{bmatrix} 0\,0 & N_{I,x} & 0\,0 & -N_I & 0 & 0\,0 \\ 0\,0 & N_{I,y} & 0\,0 & 0 & -N_I & 0\,0 \end{bmatrix};\mathbf{B}_{s1I}=\begin{bmatrix} 0\,0\,0 & N_I & 0 & 0\,0\,0\,0 \\ 0\,0\,0 & 0 & N_I & 0\,0\,0\,0 \end{bmatrix} \tag{23}$$

Substituting Eq. (20) into Eq. (10), we obtain

$$\bar{\mathbf{E}}_b=\sum_{I=1}^{m\times n}\bar{\mathbf{B}}_{b\varphi I}\mathbf{d}_I;\ \bar{\mathbf{E}}_s=\sum_{I=1}^{m\times n}\bar{\mathbf{B}}_{s\varphi I}\mathbf{d}_I;\ \bar{\mathbf{H}}_b=\sum_{I=1}^{m\times n}\bar{\mathbf{B}}_{b\psi I}\mathbf{d}_I;\ \bar{\mathbf{H}}_s=\sum_{I=1}^{m\times n}\bar{\mathbf{B}}_{s\psi I}\mathbf{d}_I \tag{24}$$

where

$$\bar{\mathbf{B}}_{b\varphi I}=\begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&-N_I&0 \end{bmatrix};\ \bar{\mathbf{B}}_{s\varphi I}=\begin{bmatrix} 0&0&0&0&0&0&0&-N_{I,x}&0 \\ 0&0&0&0&0&0&0&-N_{I,y}&0 \end{bmatrix}$$

$$\bar{\mathbf{B}}_{b\psi I}=\begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&-N_I \end{bmatrix};\ \bar{\mathbf{B}}_{s\psi I}=\begin{bmatrix} 0&0&0&0&0&0&0&0&-N_{I,x} \\ 0&0&0&0&0&0&0&0&-N_{I,y} \end{bmatrix} \tag{25}$$

Inserting Eq. (20) into Eq. (16) yields the components of displacement fields

$$\bar{\mathbf{u}}=\{\mathbf{u}_0 \quad \mathbf{u}_1 \quad \mathbf{u}_2\}^T=\sum_{I=1}^{mxn}\{\mathbf{M}_{0I} \quad \mathbf{M}_{1I} \quad \mathbf{M}_{2I}\}^T\mathbf{q}_I=\sum_{I=1}^{mxn}\bar{\mathbf{M}}_I\mathbf{q}_I \tag{26}$$

where

$$\mathbf{M}_{0I}=\begin{bmatrix} N_I&0&0&0&0&0&0&0&0 \\ 0&N_I&0&0&0&0&0&0&0 \\ 0&0&N_I&0&0&0&0&0&0 \end{bmatrix};\ \mathbf{M}_{1I}=-\begin{bmatrix} 0&0&0&0&0&N_I&0&0&0 \\ 0&0&0&0&0&0&N_I&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{bmatrix} \tag{27}$$

$$\mathbf{M}_{2I}=\begin{bmatrix} 0&0&0&N_I&0&0&0&0&0 \\ 0&0&0&0&N_I&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

The matrix $\mathbf{B}_g$ are represented as follows

$$\mathbf{B}_g=\sum_{I=1}^{m\times n}\mathbf{B}_{gI}\mathbf{d}_I \tag{28}$$

where

$$\mathbf{B}_{gI} = \begin{bmatrix} 0 & 0 & N_{I,x} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & N_{I,y} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{29}$$

The final equation is obtained

$$\left(\left(\mathbf{K} - \mathbf{K}_g\right) - \omega^2 \mathbf{M}\right)\bar{\mathbf{q}} = \mathbf{0} \tag{30}$$

where **K, M** and $\mathbf{K}_g$ are the global stiffness matrix, mass matrix and geometry matrix formulated

$$\begin{aligned}
\mathbf{K} = & \int_\Omega \left(\bar{\mathbf{B}}_b\right)^T \bar{\mathbf{D}}^b_{uu} \bar{\mathbf{B}}_b \mathrm{d}\Omega - \int_\Omega \left(\bar{\mathbf{B}}_b\right)^T \bar{\mathbf{D}}^b_{ue} \bar{\mathbf{B}}_{b\varphi} \mathrm{d}\Omega - \int_\Omega \left(\bar{\mathbf{B}}_b\right)^T \bar{\mathbf{D}}^b_{um} \bar{\mathbf{B}}_{b\psi} \mathrm{d}\Omega + \int_\Omega \left(\bar{\mathbf{B}}_s\right)^T \mathbf{D}^s_{uu} \bar{\mathbf{B}}_s \mathrm{d}\Omega \\
& - \int_\Omega \left(\bar{\mathbf{B}}_s\right)^T \bar{\mathbf{D}}^s_{ue} \bar{\mathbf{B}}_{s\varphi} \mathrm{d}\Omega - \int_\Omega \left(\bar{\mathbf{B}}_s\right)^T \bar{\mathbf{D}}^s_{um} \bar{\mathbf{B}}_{s\psi} \mathrm{d}\Omega - \int_\Omega \left(\bar{\mathbf{B}}_{b\varphi}\right)^T \bar{\mathbf{D}}^b_{eu} \bar{\mathbf{B}}_b \mathrm{d}\Omega - \int_\Omega \left(\bar{\mathbf{B}}_{b\varphi}\right)^T \bar{\mathbf{D}}^b_{ee} \bar{\mathbf{B}}_{b\varphi} \mathrm{d}\Omega \\
& - \int_\Omega \left(\bar{\mathbf{B}}_{b\varphi}\right)^T \bar{\mathbf{D}}^b_{em} \bar{\mathbf{B}}_{b\psi} \mathrm{d}\Omega - \int_\Omega \left(\bar{\mathbf{B}}_{s\varphi}\right)^T \bar{\mathbf{D}}^s_{eu} \bar{\mathbf{B}}_s \mathrm{d}\Omega - \int_\Omega \left(\bar{\mathbf{B}}_{s\varphi}\right)^T \bar{\mathbf{D}}^s_{ee} \bar{\mathbf{B}}_{s\varphi} \mathrm{d}\Omega - \int_\Omega \left(\bar{\mathbf{B}}_{s\varphi}\right)^T \bar{\mathbf{D}}^s_{em} \bar{\mathbf{B}}_{s\psi} \mathrm{d}\Omega \\
& - \int_\Omega \left(\bar{\mathbf{B}}_{b\psi}\right)^T \bar{\mathbf{D}}^b_{mu} \bar{\mathbf{B}}_b \mathrm{d}\Omega - \int_\Omega \left(\bar{\mathbf{B}}_{b\psi}\right)^T \bar{\mathbf{D}}^b_{me} \bar{\mathbf{B}}_{b\varphi} \mathrm{d}\Omega - \int_\Omega \left(\bar{\mathbf{B}}_{b\psi}\right)^T \bar{\mathbf{D}}^b_{mm} \bar{\mathbf{B}}_{b\psi} \mathrm{d}\Omega - \int_\Omega \left(\bar{\mathbf{B}}_{s\psi}\right)^T \bar{\mathbf{D}}^s_{mu} \bar{\mathbf{B}}_s \mathrm{d}\Omega \\
& - \int_\Omega \left(\bar{\mathbf{B}}_{s\psi}\right)^T \bar{\mathbf{D}}^s_{me} \bar{\mathbf{B}}_{s\varphi} \mathrm{d}\Omega - \int_\Omega \left(\bar{\mathbf{B}}_{s\psi}\right)^T \bar{\mathbf{D}}^s_{mm} \bar{\mathbf{B}}_{s\psi} \mathrm{d}\Omega \\
\mathbf{K}_g = & \int_\Omega \left(1 - \xi^2 \nabla^2\right) \left(\mathbf{B}_g\right)^T \begin{bmatrix} N_x^{elec} + N_x^{mag} & 0 \\ 0 & N_y^{elec} + N_y^{mag} \end{bmatrix} \mathbf{B}_g \mathrm{d}\Omega \\
\mathbf{M} = & \int_\Omega \left(1 - \xi^2 \nabla^2\right) \bar{\mathbf{M}}^T \mathbf{I}_m \bar{\mathbf{M}} \mathrm{d}\Omega
\end{aligned} \tag{31}$$

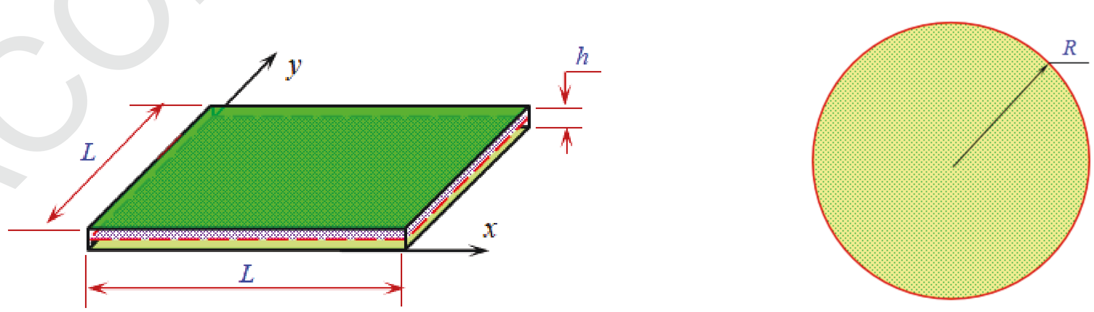
in which ω and $\bar{\mathbf{q}}$ are the natural frequency and modes shape, respectively.

5. NUMERICAL EXAMPLES

This section considers a MEE nanoplate made of $BaTi_2O_3$ and $CoFe_2O_4$. Table 2 provides the corresponding material properties. Two MEE models including square and circle nanoplates shown in Figure 2 are performed. The non-dimensional frequency is subsequently examined and derived as follows

+ Square nanoplates $$\bar{\omega} = \omega L \sqrt{\rho / c_{11e}} \tag{32}$$

+ Circle nanoplates $$\bar{\omega} = \omega \frac{R^2}{h} \sqrt{\rho / c_{11e}} \tag{33}$$



A square nanoplate A circle nanoplate

Figure 2. MEE nanoplates models.

5.1 A square MEE nanoplate

Consider a simply supported square MEE plate with length (L) and thickness (h), the influence of $e_0 = \xi / L$ on the lowest two fundamental vibration modes of the MEE nanoplate with L = 60, L/h = 15 is summarized in Table 3. The numerical outcomes obtained in this study are verified with those reported by Ke et al. [4] employing the Kirchhoff plate theory, by Gholami et al. [24] using HSDT and by Thai et al. [25] through isogeometric analysis. Overall, the results in Table 3 show excellent agreement with analytical solutions [4], the Navier's solution approach [24] and numerical method [25], demonstrating the reliability of the proposed approach. The results further show that larger values of the nonlocal parameter lead to lower natural frequencies, indicating a softening effect on the structural stiffness.

Subsequently, the first five natural frequencies of the MEE nanoplate with L/h = 10, 100 is analyzed, with the outcomes presented in Table 4. The results once again reveal an excellent correspondence with those available in the literature. In addition, it is evident that as the length-to-thickness ratio decreases, the corresponding frequencies rise, indicating a stiffening behavior in thicker plates.

Finally, the mode shapes associated with the six lowest vibration modes are depicted in Figure 3. These shapes are consistent with the expected physical deformation patterns of the nanoplate, further validating the accuracy of the current numerical model.

Table 2. Material properties of $BaTi_2O_3$-$CoFe_2O_4$.

Elastic (GPa)	$c_{11} = c_{22} = 226; c_{12} = 125; c_{13} = 124; c_{44} = c_{55} = 44.2; c_{66} = 50.5$
Piezoelectric (C/m^2)	$e_{31} = e_{32} = -2.2; e_{33} = 9.3; e_{15} = 5.8$
Dielectric (10^{-9} C/V.m)	$k_{11} = k_{22} = 5.64; k_{33} = 6.35$
Piezomagnetic (N/A.m)	$q_{15} = q_{24} = 275; q_{31} = q_{32} = 290.1; q_{33} = 349.9$
Magnetoelectric (10^{-12} Ns/VC)	$d_{11} = d_{22} = 5.367; d_{33} = 2737.5$
Magnetic (10^{-6} Ns^2/C^2)	$\mu_{11} = \mu_{22} = -297; \mu_{33} = 83.5$

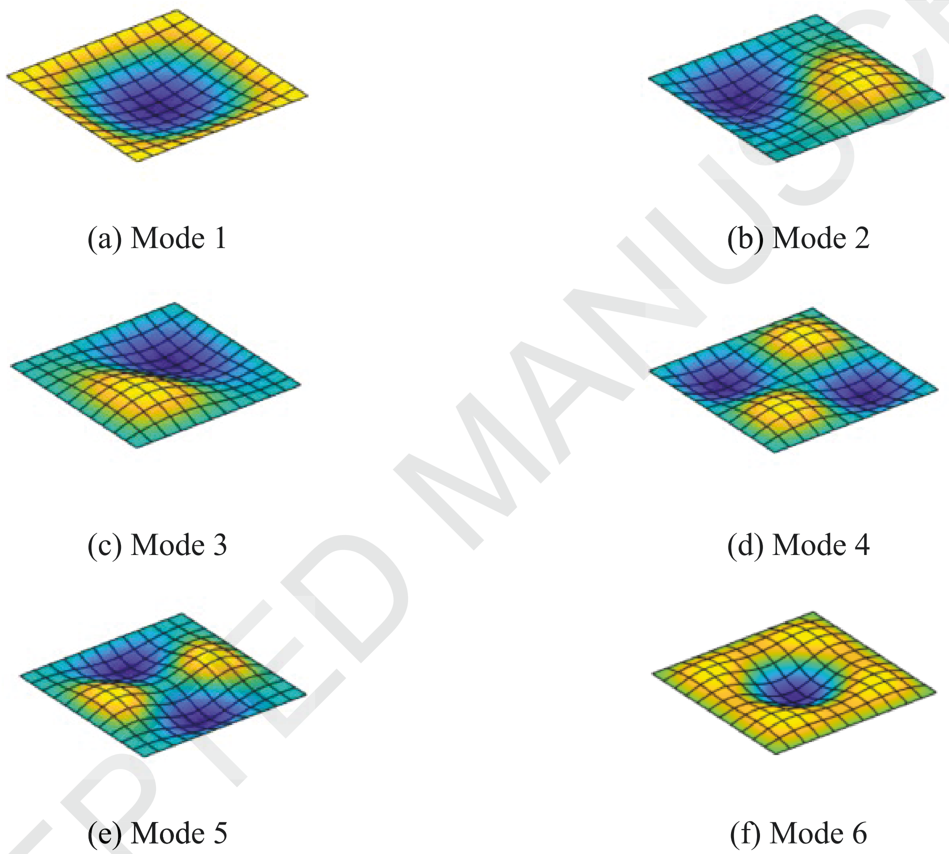
Table 3. The lowest two frequencies of the MEE nanoplates (L = 60, L/h = 15).

Method	$e_0 = 0$		$e_0 = 0.2$		$e_0 = 0.4$	
	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
Thai et al. [25]	0.3830	0.9329	0.2863	0.0541	0.1878	0.3128
Gholami et al. [24]	0.3682	0.9136	0.2753	0.5372	0.1806	0.3103
Ke et al. [4]	0.3698	0.9247	0.2764	0.5362	0.1813	0.3100
Present (p=3)	0.3830	0.9329	0.2863	0.5410	0.1878	0.3128
Present (p=5)	0.3830	0.9330	0.2863	0.5410	0.1878	0.3128

Table 4. Effects of the length-to-thickness ratio on the first five frequencies of the MEE nanoplate

L/h	e_0	Method	Mode				
			1	2	3	4	5
100	1	Thai et al. [25]	0.0128	0.0206	0.0206	0.0261	0.0293

		Present (*p*=3)	0.0128	0.0206	0.0206	0.0261	0.0293
		Present (*p*=5)	0.0128	0.0206	0.0206	0.0261	0.0293
	0.5	Thai et al. [25]	0.0240	0.0400	0.0400	0.5130	0.0578
		Present (*p*=3)	0.0240	0.0400	0.0400	0.0513	0.0578
		Present (*p*=5)	0.0240	0.040	0.0400	0.0513	0.0578
10	1	Thai et al. [25]	0.1234	0.1878	0.1878	0.2275	0.2476
		Present (*p*=3)	0.1234	0.1878	0.1878	0.2275	0.2476
		Present (*p*=5)	0.1234	0.1878	0.1878	0.2276	0.2477
	0.5	Thai et al. [25]	0.2307	0.3649	0.3649	0.4468	0.4879
		Present (*p*=3)	0.2307	0.3649	0.3649	0.4468	0.4879
		Present (*p*=5)	0.2308	0.3649	0.3649	0.4469	0.4881



(a) Mode 1 (b) Mode 2

(c) Mode 3 (d) Mode 4

(e) Mode 5 (f) Mode 6

Figure 3. The lowest six mode shapes the MEE nanoplates with L/h=10, e_0 = 0.5.

5.2 A circle MEE nanoplate

For MEE circular nanoplates with radius R, the non-dimensional natural frequency is formulated in Eq. (33). The first natural frequency of a clamped MEE circular nanoplate is computed with results provided in Table 5. The data show that larger the nonlocal parameter reduces the frequency, reflecting a decrease in effective stiffness. Conversely, increasing the radius-to-thickness ratio results in higher natural frequencies.

Subsequently, effects of both the applied electric voltage and magnetic potential on the fundamental frequency of the clamped MEE circular nanoplate with (*p*=5) are investigated, as presented in Figure 4. The results reveal opposite trends: the frequency rises as the electric voltage increases, whereas it declines when the magnetic potential becomes stronger. These trends imply that an elevated electric voltage enhances the effective stiffness of the plate, while a higher magnetic potential reduces it.

Table 5. The first six frequencies of the clamped MEE circle nanoplate with R = 10

R/h	ξ	Method	Mode					
			1	2	3	4	5	6
10	1	Present (p=3)	2.8271	5.4692	5.4692	8.2828	8.2828	9.2384
		Present (p=5)	2.8273	5.4699	5.4699	8.2844	8.2860	9.2403
	2	Present (p=3)	2.5881	4.5767	4.5767	6.3978	6.3991	7.0073
		Present (p=5)	2.5883	4.5773	4.5773	6.3990	6.4002	7.0086
	3	Present (p=3)	2.2948	3.7328	3.7328	4.9417	4.9417	5.3691
		Present (p=5)	2.2950	3.7333	3.7333	4.9426	4.9436	5.3701
50	1	Present (p=3)	2.9211	5.8138	5.8138	9.0562	9.0904	10.2078
		Present (p=5)	2.9211	5.8138	5.8138	9.0562	9.0905	10.2079
	2	Present (p=3)	2.6720	4.8553	4.8553	6.9754	7.0065	7.7267
		Present (p=5)	2.6720	4.8553	4.8553	6.9754	7.0066	7.7268
	3	Present (p=3)	2.3675	3.9550	3.9550	5.3805	5.4073	5.9178
		Present (p=5)	2.3676	3.9551	3.9551	5.3806	5.4074	5.9178

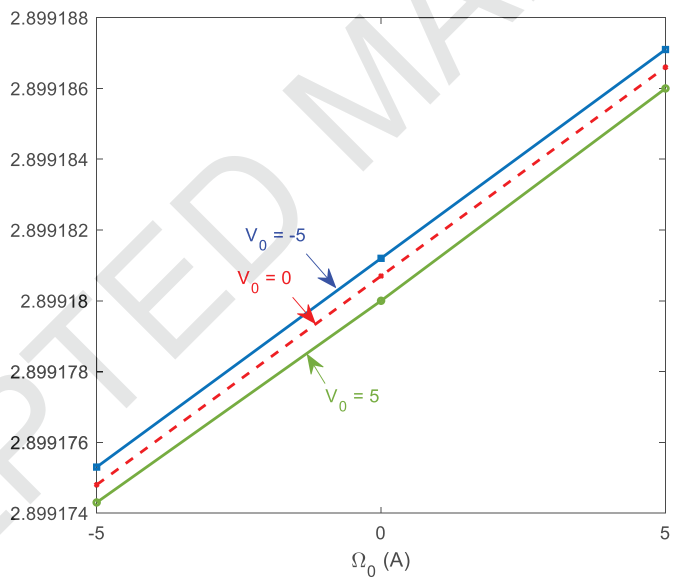


Figure 4. Effects of the magnetic potential and external electric voltage on the first frequency of the clamped MEE circle nanoplate (R=10, R/h=20, $\xi=1$).

6. CONCLUSIONS

This study presented a novel theoretical framework for MEE nanoplates by combining Eringen's theory, Chebyshev shear deformation theory and isogeometric analysis. The formulation, derived through the principle of extended virtual displacement, successfully incorporates small-scale effects while maintaining computational efficiency and accuracy. Besides, this formulation automatically enforces the zero-shear-stress condition at the plate's top and bottom surfaces, eliminating the need for any supplementary constraints. The numerical investigations demonstrate excellent agreement with reference results, confirming the reliability and robustness of the proposed model. Parametric studies reveal that increasing the nonlocal parameter leads to a noticeable reduction in natural frequencies, highlighting the softening influence of nonlocal effects on the structural stiffness of nanoplates. Conversely, decreasing the length-to-thickness ratio produces higher frequencies, indicating enhanced stiffness in thicker plates. Besides, applying an electric voltage effectively enhances the plate stiffness, whereas increasing the magnetic potential reduces it. Overall, the proposed nonlocal Chebyshev–IGA formulation provides an accurate, efficient and versatile tool for calculating and simulation of the MEE nanostructures, advanced nano-scale devices and smart material systems.

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