

# EFFECT OF INITIAL STRESS ON THE REFLECTION AND TRANSMISSION OF SHEAR HORIZONTAL WAVES AT THE INTERFACE BETWEEN TWO TRANSVERSELY ISOTROPIC ELASTIC HALF-SPACES

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**Abstract.** The phenomenon of reflection and transmission of shear wave (horizontally polarized) at an imperfect interface between two different transversely isotropic elastic half-spaces is investigated in this paper. First, the characteristic equation in the transversely isotropic half-space is derived. The forms of the incident, reflected, and transmitted waves are derived. Then, from the interface conditions for two cases (perfect and imperfect interfaces), the displacement and stress are required to be satisfied. The reflection and transmission coefficients in terms of the amplitude ratio of various waves are calculated using interface continuous conditions. The energy ratios of reflected and transmitted waves of the structures have been computed and shown to observe the law of conservation of energy. Numerical examples are shown graphically to exhibit the effects of initial stress as well as the imperfect interface on the phase velocity, phase angles, and reflection and transmission coefficients.

**Keywords:** reflection, transmission, transversely isotropic, initial stress, imperfect interface.

## 1. INTRODUCTION

The study of reflection and transmission of a plane wave at the separating interface of two different media is a very important problem in many research areas such as non-destructive testing, structural health monitoring, reflection seismology, smart material technology [1]. A comprehensive understanding of how initial stresses interact with wave propagation is essential for accurate predictions of material behavior when acted

upon by an incident wave. Biot [2] developed a definitive theory that explains the elastodynamics of a body under initial stress. An elegant and elaborate exposition of this theory is found in Biot [3]. The effect of initial stresses in the study of such phenomena in various materials was investigated by several researchers, such as Chattopadhyay et al. [4], Garg [5], Guha and Singh [6], Liu et al. [7], etc.

Transversely isotropic elastic materials are a specific class of materials that exhibit symmetry in their mechanical properties with respect to one axis. This means that while the material has identical properties in all directions within a plane perpendicular to the axis of symmetry, it can have different properties along that axis. The equations governing displacements in the  $x_1x_3$  plane for P, SV waves are completely uncoupled for SH wave [8]. Therefore we can consider separately the problem of SH wave propagation in a transversely isotropic medium. The behavior of transmission coefficients for shear horizontal (SH) wave propagation with oblique incidence in piezocomposite layered systems is investigated by Ramos et al. [9]. The transmission behavior of SH waves in a piezoelectric laminated structure is studied by Alvarez-Mesquida et al. [10]. Qian et al. [11] studied the effect of initial stress on the propagation behavior of SH-waves in multilayered piezoelectric composite structures.

In most of the above studies, the interface bonding between any two media is assumed to be perfect/welded. However, after a period of time, the perfect bonding or continuity of mechanical and electrical properties at the interface may be affected by several reasons like the aging of the applied glue at the interface, micro defects, diffusion of impurities, interface damage or defects in fabrication and corrosion of materials, etc. Imperfect bonding means that the stress components are continuous and a small displacement field is not. For an imperfect interface, the spring model of Hashin is commonly used. In this model, the normal and tangential interfacial stiffnesses characterize the properties of the imperfect interface between the two solids [12, 13]. Several authors have considered the effect of imperfect bonding on their problems, for example, Barak and Kaliraman [14], Goyal et al. [15], Huang and Rokhlin [13], Kumar et al. [16], Pang and Liu [17], Singh [18], etc. Tung [19, 20] investigated the influence of boundary conditions on the reflection and transmission of qP-wave at an interface between two nonlocal transversely isotropic elastic, liquid-saturated porous half-spaces.

The reflection and transmission of SH waves at an imperfect interface between two transversely isotropic elastic half-spaces with pre-stress effects taken into account are investigated in this paper. Moreover, the normal and shear initial stresses are considered instead of only considering the normal initial stress in most of the literature.

## 2. BASIC EQUATIONS AND FORMULATION OF THE PROBLEM

To consider the SH wave propagating in the  $x_1$ -direction and causing displacement in the  $x_2$ -direction only, its displacement component is given by

$$u_2 = u_2(x_1, x_3), \quad u_1 = u_3 = 0. \quad (1)$$

The constitutive relations for a homogeneous transversely isotropic elastic medium can be written as [8, 21]

$$\sigma_{21} = c_{66}u_{2,1}, \quad \sigma_{23} = c_{44}u_{2,3}, \quad (2)$$

where  $\sigma_{21}, \sigma_{23}$  are the components of stress and  $c_{44}, c_{66}$  are elastic constants of the transversely isotropic material.

The mechanical governing equation with initial stress (without body forces) considered can be expressed as [7, 21, 22]

$$\sigma_{ij,i} + (u_{j,k}\sigma_{ki}^0)_{,i} = \rho\ddot{u}_j, \quad (3)$$

where  $\rho$  is the mass density,  $\sigma_{ki}^0$  are components of initial stresses, the temporal derivative, spatial derivative are denoted by the superposed dot, a comma in the subscript, respectively.

Inserting (2) into (3) and taking into account (1), we have the displacement equation of motion for SH wave that propagates in the  $x_1x_3$  plane, namely

$$(c_{66} + \sigma_{11}^0)u_{2,11} + 2\sigma_{13}^0u_{2,13} + (c_{44} + \sigma_{33}^0)u_{2,33} = \rho\ddot{u}_2. \quad (4)$$

As illustrated in Fig. 1, the present research analyses the reflection and transmission phenomenon of SH wave at the interface of two transversely isotropic elastic half-spaces with interfacial imperfection. Let us assume a Cartesian coordinate system in such a way that  $x_1$ -axis lies along the separating interface of two halfspaces and  $x_3$ -axis is pointing vertically upwards. For the oblique incidence of the qSH wave from the  $\Omega^+$  medium at the interface  $x_3 = 0$ , all kinds of scattered waves are depicted in Fig. 1.

Now, considering the generalized Snell's law, which states that the apparent wave number is the same for each wave, the solution of (4) is considered as [19, 20]

$$u_2 = ae^{ik(x_1 + \xi x_3 - ct)}, \quad (5)$$

where  $k$  is the  $x_1$ -component of the wavenumber,  $c$  is phase velocity along  $x_1$ ,  $\xi$  is an unknown ratio of the wave vector components along the  $x_3$ - and  $x_1$ -directions,  $a$  is the unknown amplitude of the displacement.

With the aid of (5), Eq. (4) and the condition for a non-trivial solution in  $a$ , we obtain a two-degree polynomial equation for  $\xi$ , namely

$$t_2\xi^2 + t_1\xi + t_0 = 0, \quad (6)$$

where

$$t_2 = c_{44} + \sigma_{33}^0, \quad t_1 = 2\sigma_{13}^0, \quad t_0 = c_{66} + \sigma_{11}^0 - \rho c^2. \quad (7)$$

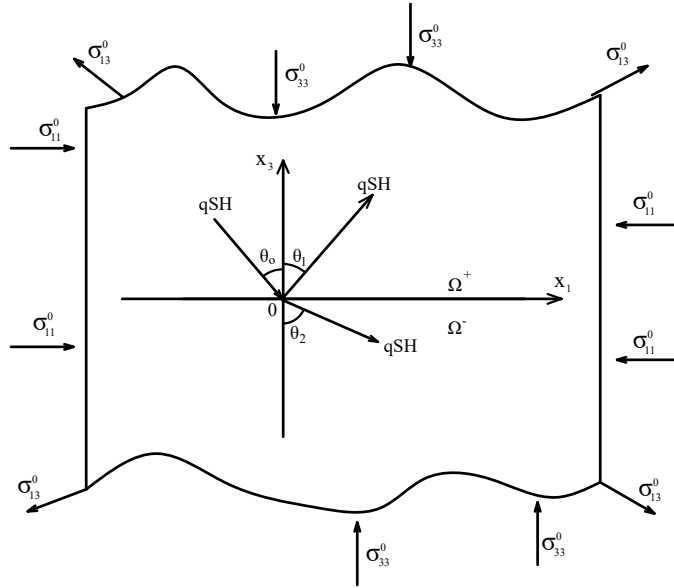


Fig. 1. Geometry of the problem

From the characteristic equation (6), it can be seen that there are two quasi-transverse (qSH) waves that propagate in opposite directions in the medium. To find the expression of the velocity of qSH wave in this medium, we consider a plane wave with phase velocity  $v$  in the direction making an angle  $\theta$  with the vertical axis that is expressed by

$$u_2 = ae^{ik_0(p_1x_1 + p_3x_3 - vt)}, \quad (8)$$

where  $p_1 = \sin \theta$ ,  $p_3 = -\cos \theta$  are components of the propagation unit vector. It is noted that  $k_0 = \frac{k}{p_1}$  and  $v = p_1c$ . Substituting (8) into (4), we obtain a real root  $v$  corresponding to the speed of qSH wave propagating in the medium.

$$v = \sqrt{\frac{(c_{66} + \sigma_{11}^0)p_1^2 + 2\sigma_{13}^0p_1p_3 + (c_{44} + \sigma_{33}^0)p_3^2}{\rho}}. \quad (9)$$

### 3. BOUNDARY CONDITIONS

To investigate the influence of the imperfect interface on the reflection and transmission of waves at the separating interface of two halfspaces, we consider two distinct types of interface at  $x_3 = 0$ .

(i) Perfect interface: In case of perfect interface, the stresses and displacement are continuous at the interface [23, 24].

$$u_2^+ = u_2^-, \quad \sigma_{23}^+ + \sigma_{13}^0 u_{2,1}^+ + \sigma_{33}^0 u_{2,3}^+ = \sigma_{23}^- + \sigma_{13}^0 u_{2,1}^- + \sigma_{33}^0 u_{2,3}^-. \quad (10)$$

(ii) Imperfect interface: In this case, the spring model is introduced [12, 13, 25]. The displacements are discontinuous at the interface [21, 26, 27].

$$\begin{aligned} \sigma_{23}^+ + \sigma_{13}^0 u_{2,1}^+ + \sigma_{33}^0 u_{2,3}^+ &= k_T (u_2^+ - u_2^-), \\ \sigma_{23}^+ + \sigma_{13}^0 u_{2,1}^+ + \sigma_{33}^0 u_{2,3}^+ &= \sigma_{23}^- + \sigma_{13}^0 u_{2,1}^- + \sigma_{33}^0 u_{2,3}^-, \end{aligned} \quad (11)$$

where  $k_T$  denotes the interface parameter characterizing the interfacial imperfection and it has the dimension of N/m<sup>3</sup>.

#### 4. THE REFLECTION, TRANSMISSION COEFFICIENTS

From the characteristic equation (6), the incident qSH wave at the interface will generate the reflected qSH wave in half-space  $\Omega^+$  and the transmitted qSH wave in the  $\Omega^-$  as shown in Fig. 1.

A qSH wave is incident at the interface  $x_3 = 0$  from the upper half-space  $\Omega^+$ . Based on (5), the displacement of the incident wave can be given as

$$u_2^0 = a_0 e^{ik(x_1 + \xi_0 x_3 - ct)}. \quad (12)$$

After reflecting at the interface  $x_3 = 0$ , the reflected qSH wave propagates in half-space  $\Omega^+$  and its displacement has the form

$$u_2^1 = a_1 e^{ik(x_1 + \xi_1 x_3 - ct)}. \quad (13)$$

The transmitted qSH wave propagates in half-space  $\Omega^-$  as shown by

$$u_2^2 = a_2 e^{ik(x_1 + \xi_2 x_3 - ct)}. \quad (14)$$

Now, in view of Eqs. (12) to (14), the interface conditions given by (10) reduce to the following form

$$\begin{cases} -a_1 + a_2 = a_0, \\ -((c_{44}^+ + \sigma_{33}^0)\xi_1 + \sigma_{13}^0)a_1 + ((c_{44}^- + \sigma_{33}^0)\xi_2 + \sigma_{13}^0)a_2 = ((c_{44}^+ + \sigma_{33}^0)\xi_0 + \sigma_{13}^0)a_0. \end{cases} \quad (15)$$

Similarly, substituting (12)–(14) into the boundary conditions (11), we get two equations

$$\begin{cases} \left( (c_{44}^+ + \sigma_{33}^0)i\xi_1 + \sigma_{13}^0 i - \frac{k_T}{k} \right) a_1 + \frac{k_T}{k} a_2 = \left( \frac{k_T}{k} - (c_{44}^+ + \sigma_{33}^0)i\xi_0 - \sigma_{13}^0 i \right) a_0, \\ -((c_{44}^+ + \sigma_{33}^0)\xi_1 + \sigma_{13}^0)a_1 + ((c_{44}^- + \sigma_{33}^0)\xi_2 + \sigma_{13}^0)a_2 = ((c_{44}^+ + \sigma_{33}^0)\xi_0 + \sigma_{13}^0)a_0. \end{cases} \quad (16)$$

Eqs. (15), (16) give the reflection, transmission coefficients (RTCs) which are defined by the ratio of the reflected/transmitted amplitudes to the incident amplitude for perfect and imperfect interfaces, respectively.

$$R = \frac{a_1}{a_0}, \quad T = \frac{a_2}{a_0}. \quad (17)$$

The reflection and transmission of SH waves at imperfect interfaces have wide-ranging real-world applications, including non-destructive testing, geophysical exploration, earthquake engineering, material characterization, acoustic microscopy, and guided wave testing. These applications leverage the unique properties of SH waves to detect and characterize imperfections at interfaces, providing critical insights into the integrity and performance of materials and structures.

## 5. VALIDATION

We can see that the reflection and transmission coefficients in terms of the amplitude ratio of various waves are calculated by the use of interface continuous conditions. To validate directly the numerical results with consideration of energy conservation, the energy flux reflection and the transmission coefficients are calculated instead of the displacement amplitude ratio.

Now we compute the distribution of energy among the reflected and transmitted waves due to the incidence of qSH wave at its separating interface. They should be checked to ensure that the energy of incident waves is equal to the energy sum of reflection and transmission waves. The scalar product of the surface traction and particle velocity per unit area denoted by  $P_i$  represents the rate at which energy is communicated per unit area at the surface  $x_3 = 0$ . This is given by [21, 23, 28]

$$P_i = -\dot{u}_j \sigma_{ji} - \sigma_{ik}^0 u_{j,k} \dot{u}_j. \quad (18)$$

In one period, the mathematical expression for averaged energy flux is given by

$$P_i^* = \frac{1}{2} \text{Re}(-\dot{u}_j^* \sigma_{ji} - \sigma_{ik}^0 u_{j,k} \dot{u}_j^*), \quad (19)$$

where asterisks(\*) appearing above the notations denote the complex conjugate of the derivative of the displacement components.

We may obtain the expressions of average energy flux  $E^I, E^{rqSH}, E^{tqSH}$  of the incident qSH wave, reflected qSH and transmitted qSH waves, respectively, through the unit area perpendicular to the propagation direction of the said waves for the pre-stressed medium.

$$\begin{aligned}
E^I &= \frac{1}{2}k^2c[c_{44}^+\xi_0 + \sigma_{13}^0 + \sigma_{33}^0]a_0^2, & E^{rqSH} &= \frac{1}{2}k^2c[c_{44}^+\xi_1 + \sigma_{13}^0 + \sigma_{33}^0]a_1^2, \\
E^{tqSH} &= \frac{1}{2}k^2c[c_{44}^-\xi_2 + \sigma_{13}^0 + \sigma_{33}^0]a_2^2.
\end{aligned} \tag{20}$$

Denoting  $E_r = \frac{E^{rqSH}}{E^I}$ ,  $E_t = \frac{E^{tqSH}}{E^I}$ . The expressions of all energy ratios defined above must satisfy the condition

$$E = E_r + E_t = 1. \tag{21}$$

## 6. NUMERICAL CALCULATION AND DISCUSSION

To illustrate the theoretical results obtained, numerical calculations were performed. For convenience in the numerical analysis, we introduce the dimensionless quantities

$$\begin{aligned}
f_1^+ &= \frac{c_{66}^+}{c_{44}^+}, & g_1^+ &= \frac{\sigma_{11}^0}{c_{44}^+}, & g_2^+ &= \frac{\sigma_{13}^0}{c_{44}^+}, & g_3^+ &= \frac{\sigma_{33}^0}{c_{44}^+}, & f_0 &= \frac{c_{55}^-}{c_{55}^+}, \\
f_1^- &= \frac{c_{66}^-}{c_{44}^-}, & g_1^- &= \frac{\sigma_{11}^0}{c_{44}^-}, & g_2^- &= \frac{\sigma_{13}^0}{c_{44}^-}, & g_3^- &= \frac{\sigma_{33}^0}{c_{44}^-}, & f_T &= \frac{k_T}{kc_{55}^+}.
\end{aligned} \tag{22}$$

The values of relevant elastic parameters for two half-spaces are taken following [15]:

- For half-space  $\Omega^+$ : Material constants for *AlN* are as follows:

$$c_{44}^+ = 1.18 \times 10^{11} \text{ Nm}^{-2}, \quad c_{66}^+ = 3.95 \times 10^{11} \text{ Nm}^{-2}, \quad \rho^+ = 3.62 \times 10^3 \text{ kgm}^{-3}.$$

- For half-space  $\Omega^-$ : Material constants for *BaTiO<sub>3</sub>* are given as:

$$c_{44}^- = 0.453 \times 10^{11} \text{ Nm}^{-2}, \quad c_{66}^- = 1.62 \times 10^{11} \text{ Nm}^{-2}, \quad \rho^- = 5.8 \times 10^3 \text{ kgm}^{-3}.$$

Besides, the values of the initial stresses are given as  $\sigma_{11}^0 = 0.4 \times 10^{11} \text{ Nm}^{-2}$ ,  $\sigma_{13}^0 = -0.1 \times 10^{11} \text{ Nm}^{-2}$ ,  $\sigma_{33}^0 = 0.8 \times 10^{11} \text{ Nm}^{-2}$ , and the  $x_1$ -component of the wavenumber  $k$  is fixed at 1. The tangential spring constant is such that  $k_T = 2.0$  in the imperfect interface case. In the perfect interface case,  $k_T = 2^{100}$ .

To investigate the effects of initial stress, the reflection and transmission angles as well as the reflection and transmission coefficients are computed for different incident angles. The numerical results with initial stress and without initial stress are both shown in one figure to facilitate the comparison. Specifically, the ones of waves in the presence and absence of initial stress are illustrated by the solid line and dashed line, respectively.

The effect of the initial stress on the dimensionless velocities of qSH waves is plotted in Fig. 2. It can be seen from this figure that the velocities in the medium with the initial stress are greater than those in the medium without the initial stress.

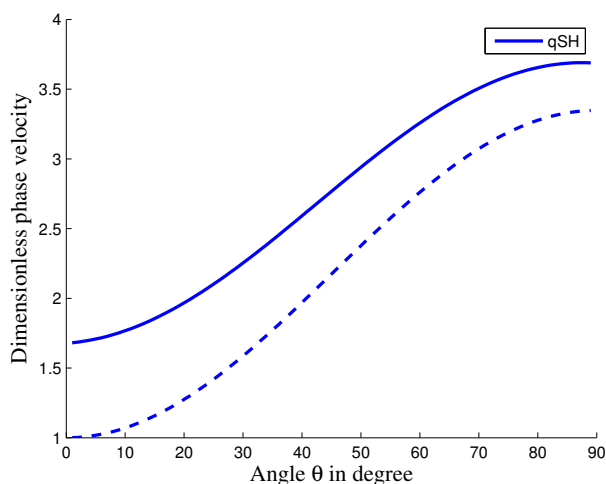


Fig. 2. Velocities of waves for presence and absence of the initial stress

Fig. 3 shows the effect of the initial stress on the RTCs in the case of the perfect interface. The RTCs of the reflected and transmitted waves are affected by the initial stress. Contrary to this, in the case of the imperfect interface, the RTCs are unaffected by the initial stress. It can be seen that the total reflection phenomenon appears in this case (see Fig. 4). The effect of initial stress on SH wave reflection and transmission is critical in fields like seismology, geophysics, non-destructive testing, and structural engineering. Initial stress alters the reflection and transmission coefficients of SH waves at interfaces. These coefficients depend on the impedance contrast between the two media, which is influenced by the initial stress. Understanding and accounting for the influence of initial

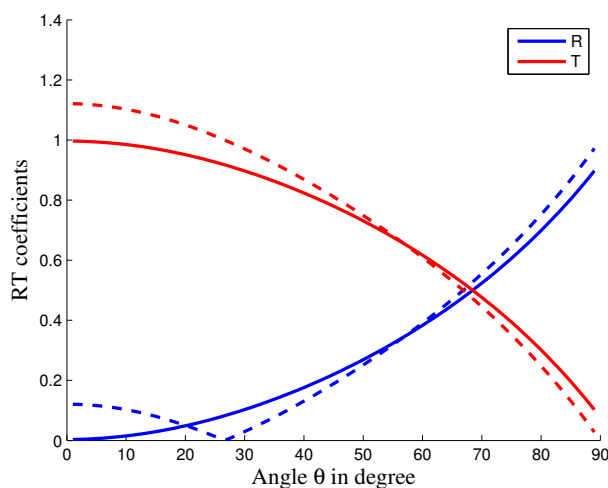


Fig. 3. The effect of the initial stress on the RTCs for the case of the perfect interface

stress is essential for accurate analysis and interpretation of SH wave behavior at interfaces in real-world scenarios.

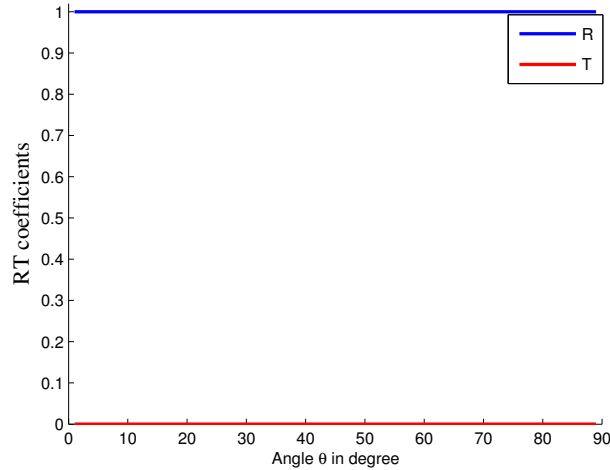


Fig. 4. The effect of the initial stress on the RTCs for the case of the imperfect interface

Fig. 5 is plotted to describe the influence of distinct common interfaces on RTCs. The comparative study of the RTCs curves suggests that the reflection coefficient  $R$  is dominant for the imperfect interface ( $k_T = 2$ ) and the transmission coefficients  $T$  for this case are negligibly small in the whole range of incident angles, while the transmission coefficient  $T$  is quite large for the perfect interface ( $k_T \rightarrow \infty$ ). Moreover, the corresponding curves of RTCs for the imperfect interface (Fig. 5(b)) and perfect interface (Fig. 3) cases are coincident in the range  $1^\circ$ – $89^\circ$ . This confirms the correctness of numerical calculations.

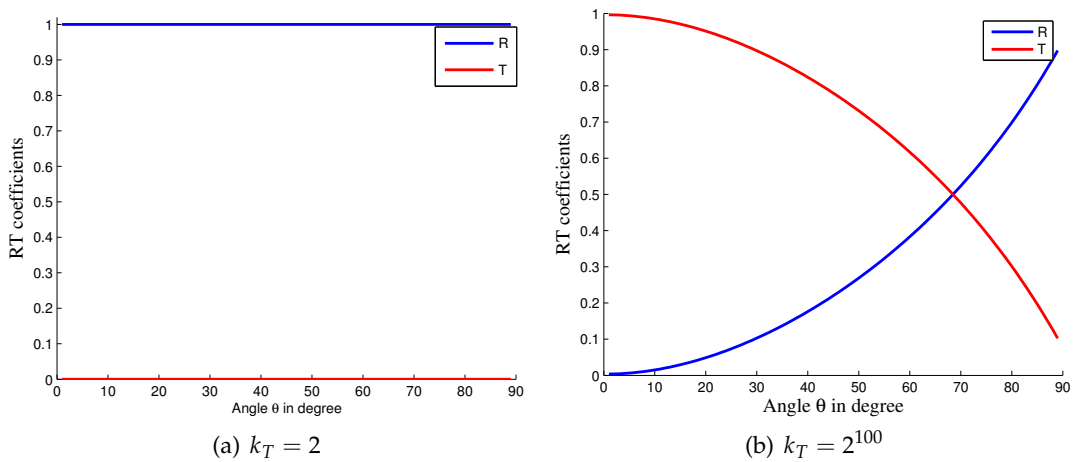


Fig. 5. The RTCs of waves for distinct common interfaces

Fig. 6 depicts the variation of energy ratios of reflected and transmitted waves with the angle of incidence for the presence and absence of initial stress. The ratios  $E_t$  ( $E_r$ ) in the medium with the initial stress are greater (lower) than those in the medium without the initial stress in the range  $60^\circ$ – $89^\circ$ . Fig. 6 once again confirms that the computational results in this section are fully reliable.

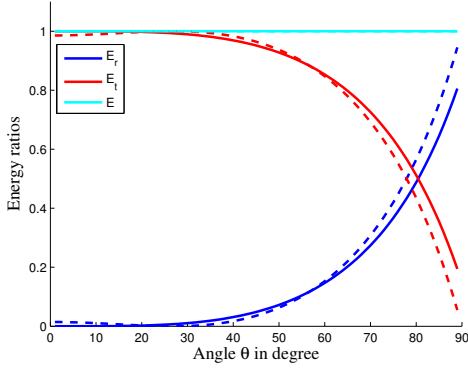


Fig. 6. Energy ratios

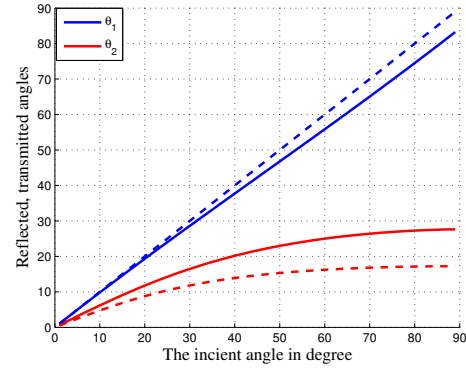
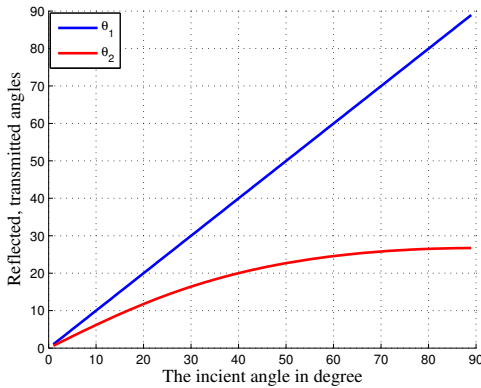
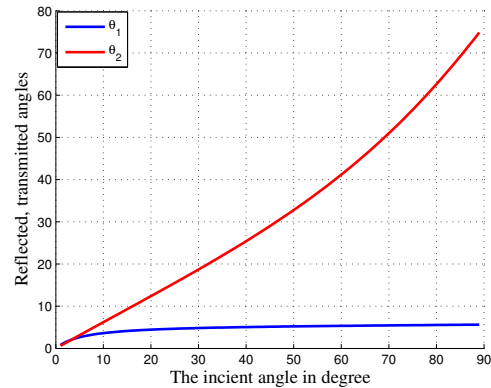


Fig. 7. Phase angles for presence and absence of the initial stress

It is assumed that the wave propagation direction deviates from the positive  $x_3$  axis with angle  $\theta$ , then  $Real(\xi) = \cot(\theta)$ , which will be used to compute the reflection and transmission angles. Fig. 7 shows the effects of initial stress on the reflection and transmission angles. It is found that the existence of the initial stress makes the transmission angle increase while the reflection angle decreases. Moreover, the value of the angle of reflection will not be equal to the value of the angle of incidence when the initial stress



(a)  $\sigma_{13}^0 = -0.01 \times 10^{11} \text{ Nm}^{-2}$



(b)  $\sigma_{13}^0 = -100 \times 10^{11} \text{ Nm}^{-2}$

Fig. 8. Effects of initial stress  $\sigma_{13}^0$  on the reflection and transmission angles

appears in the medium. It is interesting that when the values of  $\sigma_{11}^0$  and  $\sigma_{33}^0$  remain unchanged while the value of  $\sigma_{13}^0$  decreases by a factor of 100, the reflection angle is always equal to the incident angle in the range of incident angles  $1^\circ$ – $89^\circ$  (see Fig. 8(a)). When the value of  $\sigma_{13}^0$  increases by a factor of 100, the transmission angles are significantly greater than the reflection angles (see Fig. 8(b)). This shows that the value of the initial stress  $\sigma_{13}^0$  has an important influence on the propagation of the qSH wave.

## 7. CONCLUSIONS

In conclusion, a mathematical study of the effects of the initial stress on the reflection and transmission coefficients at an imperfect interface separating two transversely isotropic elastic solid half-spaces is made when the qSH wave is incident. Some important conclusions are given below:

(i) For the incident qSH wave, the expressions for reflection and transmission coefficients as well as the energy ratios of waves in the imperfect/perfect cases are given.

(ii) The effect of the initial stress on RTCs is found for the imperfect and perfect interfaces.

(iii) The dependence of the dimensionless velocities of the incident waves as well as the reflection and transmission angles of reflected and transmitted waves on the incident angle for the presence and absence of initial stress is illustrated graphically.

The initial stress has a significant effect on the SH wave propagation in transversely isotropic structures through the modified phase velocity. This makes it an essential parameter that should be considered when designing semiconductor devices. The results obtained are fundamentally useful when the initial stress is involved. Moreover, our formulas can provide some fundamental insights for deriving RTCs formulas on the welded and non-welded interfaces between two more complex media. These results can be recorded and used as the input data of the inverse problem (nondestructive evaluation of materials).

## DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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